Name:_____

Physics I Mechanics
Physics II Electricity & Magnetism

FOUNDATIONAL MATHEMATICS' SKILLS OF PHYSICS

Do not use this packet as scratch paper. Once you have worked out the problems, show your work and final answers in a concise, clear manner in this packet. All concepts from this assessment are algebra, geometry, and pre-calculus concepts that you must know.

†Denotes additional assessments or information for the Physics II E&M class only.

†Graphing Calculator Recommendation: TI-89

Symbolic Manipulations

I. Equation Derivations

Directions: Using the following 3 equations (1-1, 2-1, & 3-1), solve for a the specified variable and substitute it into another equation to derive a new equation.

$$\bar{v} = \frac{x - x_0}{t} \tag{1-1}$$

$$a = \frac{v - v_0}{t} \tag{2-1}$$

$$\bar{v} = \frac{v_0 + v}{2} \tag{3-1}$$

1. Solve equation 1-1 for x. This will now be denoted as equation 1-2.

(1-2)

2. Solve equation 2-1 for v. This is equation 2-2.

(2-2)

3. Solve equation 2-1 for v_0. This is equation 2-3.

(2-3)

4. Solve equation 2-1 for t. This is equation 2-4.

(2-4)

5. Substitute equation 3-1 directly in for $\bar{v}$ into equation 1-2. This is equation 4-1.

(4-1)

6. Substitute equation 2-2 directly in for v into equation 4-1. This is equation 5-1.

(5-1)

7. Substitute equation 2-3 directly in for v_0 into equation 4-1. This is equation 6-1.

(6-1)

8. Substitute equation 2-4 directly in for t into equation 4-1. This is equation 7-1.

(7-1)

9. Solve equation 7-1 for v^2. This is equation 7-2.

(7-2)

II. Symbolic Manipulation

1. Using the following equations, solve for a in terms of g, m_1, m_2, μ, and θ.

$$m_1 g - T = m_1 a$$
$$T - \mu N - m_2 g \sin\theta = m_2 a$$
$$N - m_2 g \cos\theta = 0$$

2. Using the following equations, solve for a in terms of g, m_1, m_2, μ_1, μ_2, θ_1, and θ_2.

$$m_1 g \sin\theta_1 - T - \mu_1 n = m_1 a$$
$$n - m_1 g \cos\theta_1 = 0$$
$$T - \mu_2 N - m_2 g \sin\theta_2 = m_2 a$$
$$N - m_2 g \cos\theta_2 = 0$$

3. Using the following equations, solve for μ in terms of θ.

$$N - mg\cos\theta = 0$$
$$mg\sin\theta - \mu N = 0$$

4. Using the following equations, solve for v in terms of g, r, and θ.

$$N\cos\theta - mg = 0$$
$$N\sin\theta = m\frac{v^2}{r}$$

:

5. Using the following equations, solve for f in terms of g, m, r, v, and θ.

$$N\cos\theta - f\sin\theta = mg$$
$$N\sin\theta + f\cos\theta = m\frac{v^2}{r}$$

6. Using the following equations, solve for μ in terms of g, r, v, and θ.

$$N\cos\theta - \mu N\sin\theta = mg$$
$$N\sin\theta + \mu N\cos\theta = m\frac{v^2}{r}$$

III. Symbolic Reduction

1. For the following equation, set $v = 0$ and solve for t.

$$v = v_0 + at$$

2. For the following equation, set $v_0 = 0$ and solve for a.

$$v_0 = v - at$$

3. For the following equation, set $v_0 = 0$ and solve for t.

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

4. For the following equation, set $x_0 = 0$ and solve for t.

$$x = x_0 + vt - \tfrac{1}{2} a t^2$$

5. For the following equation, set $x_0 = 0$ and solve for v_0.

$$v^2 = v_0^2 + 2a\left(x - x_0\right)$$

6. For the following equations, set $x_0 = 0,\ a = 0,\ y_0 = 0$, and solve for θ.

$$x = x_0 + v_{0x}t + \tfrac{1}{2}at^2$$
$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$
$$\tan\theta = \frac{v_{0y}}{v_{0x}}$$

7. For the following equations, set $x_0 = 0,\ a = 0,\ y_0 = 0$, and solve for y in terms of $g,\ \mathrm{v}_0,\ x,$ and θ.

$$x = x_0 + \left(v_0\cos\theta\right)t + \tfrac{1}{2}at^2$$
$$y = y_0 + \left(v_0\sin\theta\right)t - \tfrac{1}{2}gt^2$$

8. For the following equations, set $a = 0$ and solve for m_1 in terms of m_2, μ, and θ.

$$T - m_1 g = m_1 a$$
$$m_2 g \sin\theta - \mu N - T = m_2 a$$
$$N - m_2 g \cos\theta = 0$$

9. For the following equations, set $\theta = 0°$, and solve for a in terms of g, m_1, m_2, μ, and θ.

$$T - m_1 g = m_1 a$$
$$m_2 g \sin\theta - \mu N - T = m_2 a$$
$$N - m_2 g \cos\theta = 0$$

10. For the following equations, set $\theta = 90°$, and solve for a in terms of g, m_1, and m_2.

$$T - m_1 g = m_1 a$$
$$m_2 g \sin\theta - \mu N - T = m_2 a$$
$$N - m_2 g \cos\theta = 0$$

11. For the following equations, set $a = 0,\ \theta = 0°$, and solve for m_1 in terms of m_2 and μ.

$$T - m_1 g = m_1 a$$
$$m_2 g \sin\theta - \mu N - T = m_2 a$$
$$N - m_2 g \cos\theta = 0$$

12. For the following equations, set $a = 0,\ \theta = 90°$, and solve for m_1 in terms of m_2.

$$T - m_1 g = m_1 a$$
$$m_2 g \sin\theta - \mu N - T = m_2 a$$
$$N - m_2 g \cos\theta = 0$$

13. For the following equations, set $\theta = 0°$, and solve for f in terms of $m,\ r,$ and v.

$$f = m\left(\frac{v^2}{r}\cos\theta - g\sin\theta\right)$$

14. For the following equations, set $\theta = 0°$, and solve for μ in terms of $g,\ r,$ and v.

$$\mu = \frac{v^2\cos\theta - rg\sin\theta}{v^2\sin\theta + rg\cos\theta}$$

Algebra Review

IV. Algebraically solving for multiple unknowns.

1. Given the following equations, solve for a numeric value for θ.

$$x = y$$

$$x = \frac{v_0^2}{g}\sin(2\theta)$$

$$v_0^2 \sin^2\theta - 2gy = 0$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

2. Given the following equations, solve for a numeric value for x and y.

$$x + 2y = 32$$

$$11x + 19y = 121$$

3. Given the following equations, solve for a numeric value for x, y, and z.

$$x + 2y = 3z$$

$$3x + 2y + z = 14$$

$$4x + 5y + 6z = 42$$

V. †Matrices

Directions: Set up a matrix for the following and use the row reduced echelon form, *rref()*, function on your calculator to calculate the unknown values.

1. Given the following equations, solve for a numeric value for x, y, and z.

$$4x + 3y = 3z$$
$$3x + 5y + 7z = 241$$
$$4x + 5y + 6z = 223$$

2. Given the following equations, solve for a numeric value for x, y, and z.

$$x + y = z$$
$$2x + 3y = 16$$
$$2y + 3z = 10$$

3. Given the following equations, solve for a numeric value for $a,\ b,\ c,$ and d.

$$a + 2b + 3c = 14$$
$$4a + 3b + 2c - d = 24$$
$$15a + 13b = 114$$
$$23d - 19c = 142$$

4. Given the following equations, solve for a numeric value for $I_1,\ I_2,\ I_3,\ I_4,$ and I_5.

$$I_2 + I_3 = I_1$$
$$I_4 + I_5 = I_2$$
$$2I_1 + 2I_2 + 3I_4 = 123$$
$$2I_1 + 2I_2 + I_5 = 123$$
$$2I_1 + I_3 = 123$$

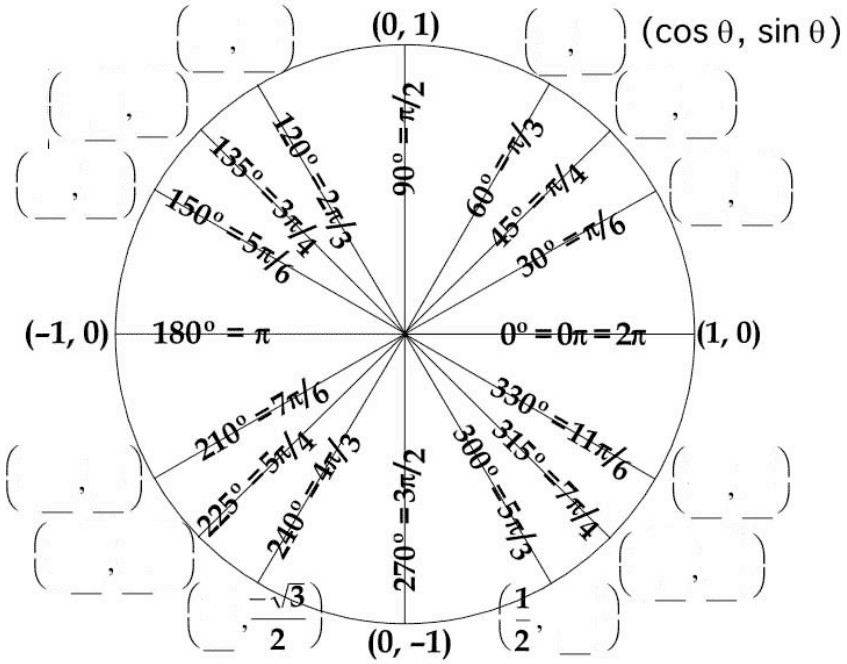
5. Given the following equations, solve for a numeric value for $I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8, I_9, I_{10}, I_{11}, I_{12},$ and I_{13}.

$$I_2 + I_5 + I_9 = I_1$$
$$I_3 + I_6 = I_2$$
$$I_4 + I_8 = I_5$$
$$I_5 + I_{13} = I_9$$
$$I_6 + I_{10} = I_{11}$$
$$I_3 + I_4 = I_7$$
$$I_8 + I_{13} = I_{12}$$
$$2I_2 + 2I_3 + 2I_7 = 156$$
$$2I_2 + 2I_6 + 2I_{11} = 156$$
$$2I_4 + 2I_5 + 2I_7 = 156$$
$$2I_5 + 2I_8 + 2I_{12} = 156$$
$$2I_9 + 2I_{12} + 2I_{13} = 156$$
$$2I_9 + 2I_{10} + 2I_{11} = 156$$

Geometry/Trigonometry Review

VI. Geometry/Trigonometry Review

1. Fill in the missing components of the Unit Circle



2. Determination of trigonometric functions (in degrees): *Place answers in fraction form*

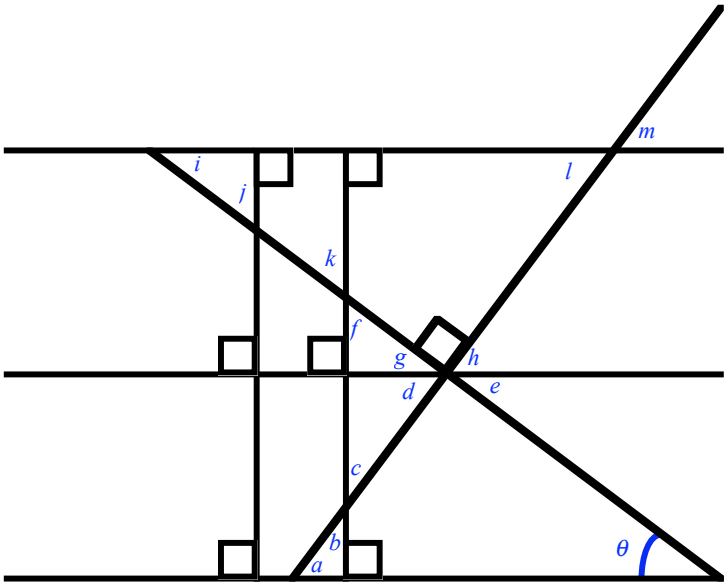
Function	0°	45°	60°	120°	135°	210°	225°	315°	330°
$\sin\theta$									
$\cos\theta$									
$\tan\theta$									

3. Determination of trigonometric functions (in radians): *Place answers in fraction form*

Function	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	$\frac{5\pi}{4}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\sin\theta$									
$\cos\theta$									
$\tan\theta$									

VII. Congruent Angles

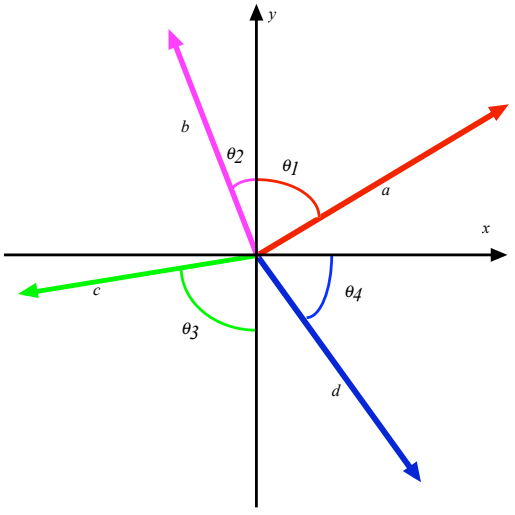
Directions: For the following diagram, check off all of the angles (*a*-*m*) below that have the same value as θ.



_____ *a* _____ *b* _____ *c* _____ *d* _____ *e*
_____ *f* _____ g _____ *h* _____ *i* _____ *j*
_____ *k* _____ *l* _____ *m*

VIII. Determination of lengths along the *x*- and *y*-axes.

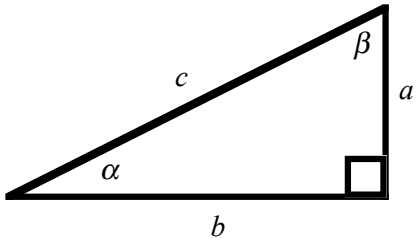
1. Given that $a = 5$, $b = 3.5$, $c = 4$, $d = 4.5$, $\theta_1 = 55°$, $\theta_2 = 20°$, $\theta_3 = 75°$, and $\theta_4 = 50°$, Calculate the distance of the lengths along the *x*- and *y*-axes for each arrow.



Arrow	*x*-axis length	*y*-axis length
a		
b		
c		
d		

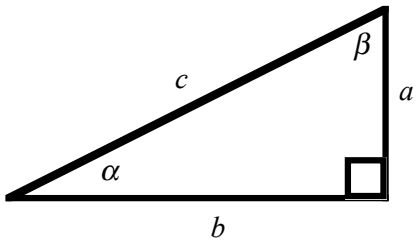
IX. Using the right triangle below (of lengths, a, b, and c, and angles, α and β) and the *Pythagorean Theorem*, write an appropriate formula to solve for each of the following:

1. a:
2. b:
3. c:



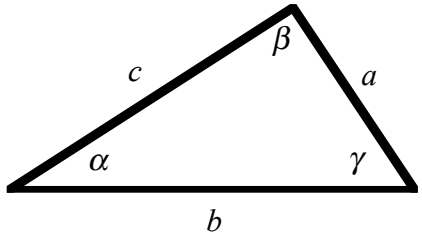
X. Using the right triangle below (of lengths, a, b, and c, and angles, α and β) and trigonometry, write an appropriate formula to solve for each of the following:

1. sin(α):
2. sin(β):
3. cos(α):
4. cos(β):
5. tan(α):
6. tan(β):



XI. Using the triangle below (of lengths, a, b, c, and angles, α, β, γ) and using the *Law of Cosines*, write an appropriate formula to solve for each of the following:

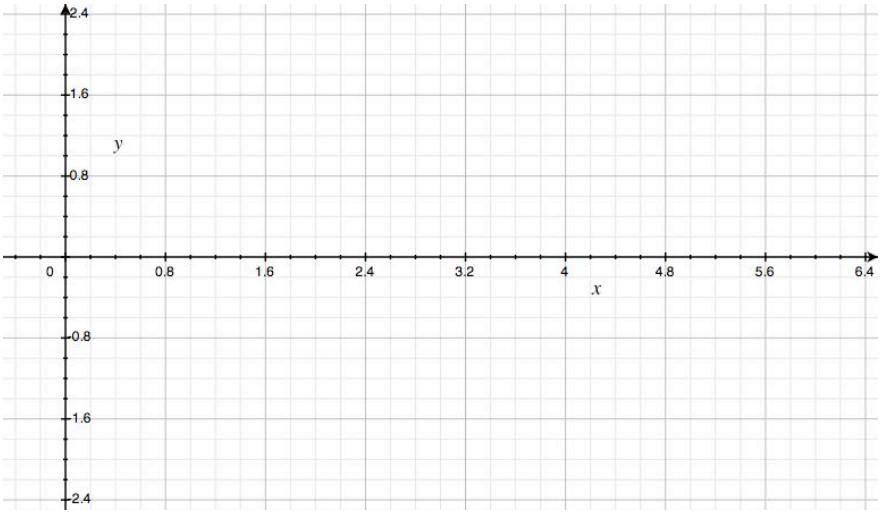
1. a:
2. b:
3. c:



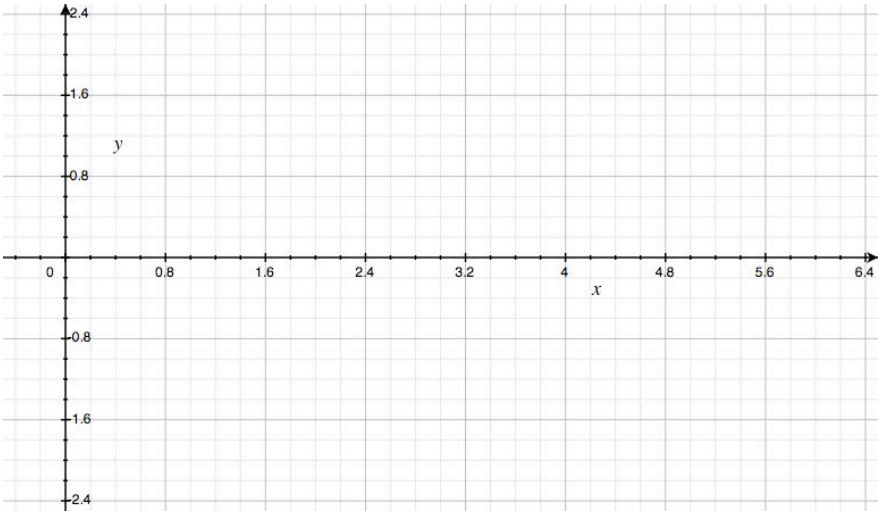
Graphing Review

XII. Graphing (Note: Computer generated graphs are also acceptable)

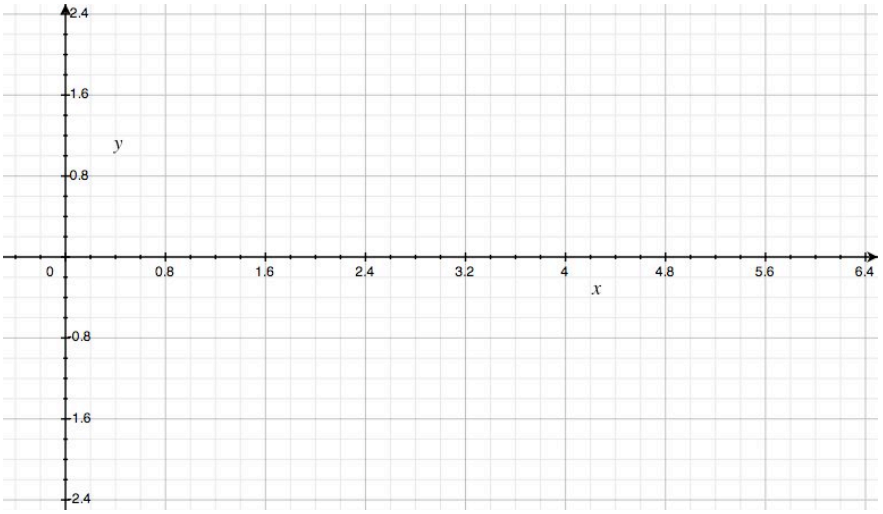
1. Group 1:
   a. $y = \frac{1}{x}$
   b. $y = -\frac{1}{x}$



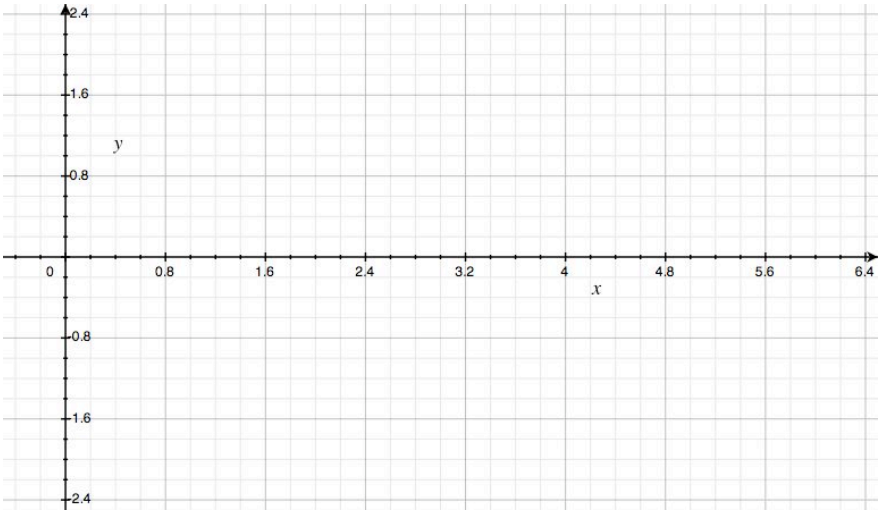
2. Group 2:
   a. $y = \frac{1}{x^2}$
   b. $y = -\frac{1}{x^2}$



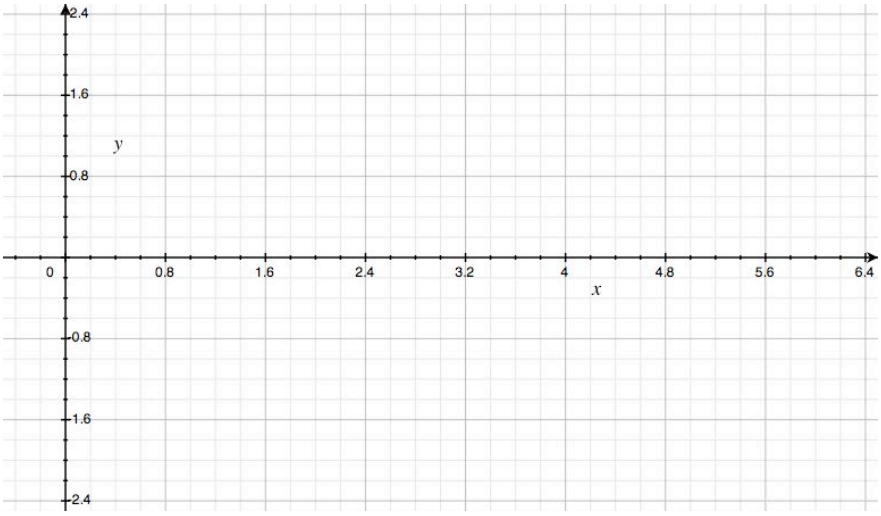
3. Group 3:
    a. $y = \ln(x)$
    b. $y = \ln\left(\frac{1}{2}x\right)$
    c. $y = \ln(2x)$



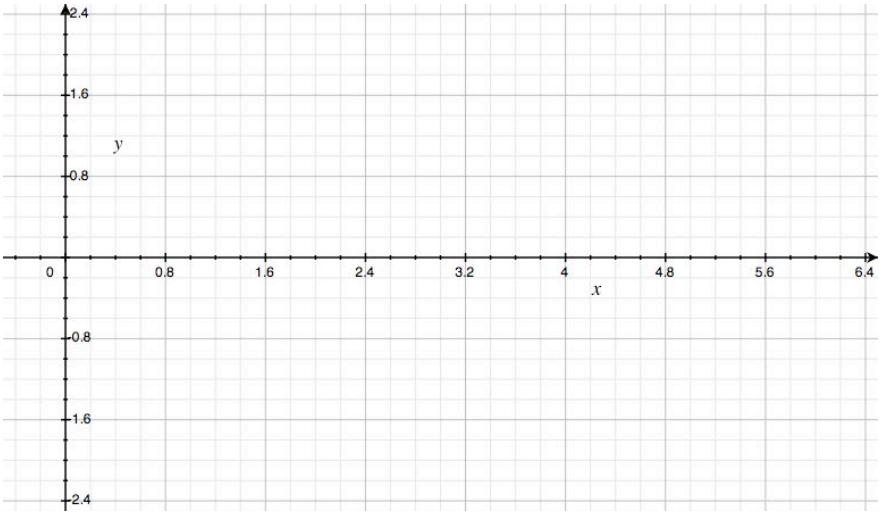
4. Group 4:
    a. $y = e^{-x}$
    b. $y = e^{-\frac{1}{2}x}$
    c. $y = e^{-2x}$



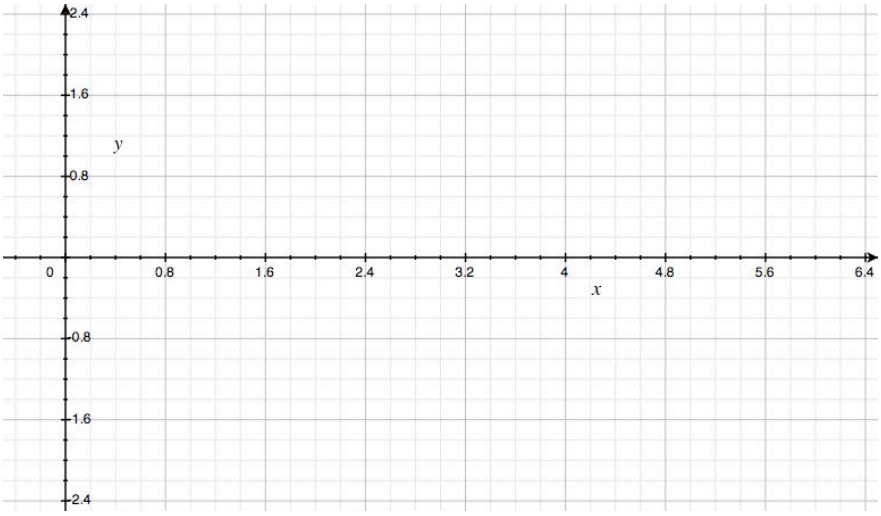
5. Group 5:
   a. $y = 1 - e^{-x}$
   b. $y = 1 - e^{-\frac{1}{2}x}$
   c. $y = 1 - e^{-2x}$



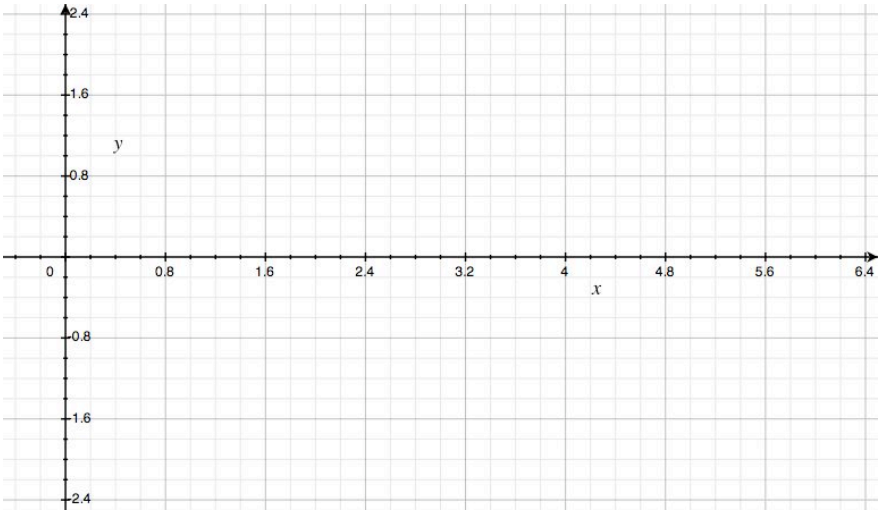
6. Group 6: (In radians from $x = 0 \rightarrow x = 2\pi$)
   a. $y = \sin(x)$
   b. $y = \sin(\frac{1}{2}x)$
   c. $y = \sin(2x)$



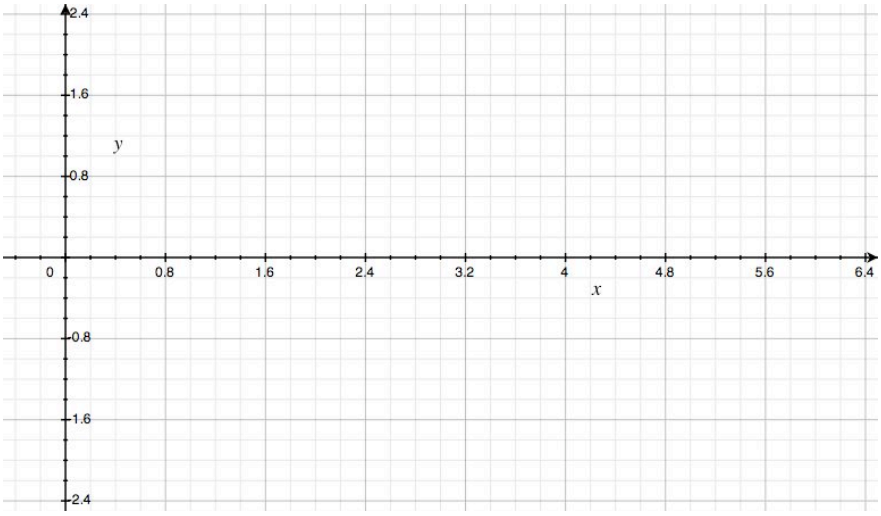
7. Group 7: (In radians from $x = 0 \rightarrow x = 2\pi$)
   a. $y = \cos(x)$
   b. $y = \cos\left(\frac{1}{2}x\right)$
   c. $y = \cos(2x)$



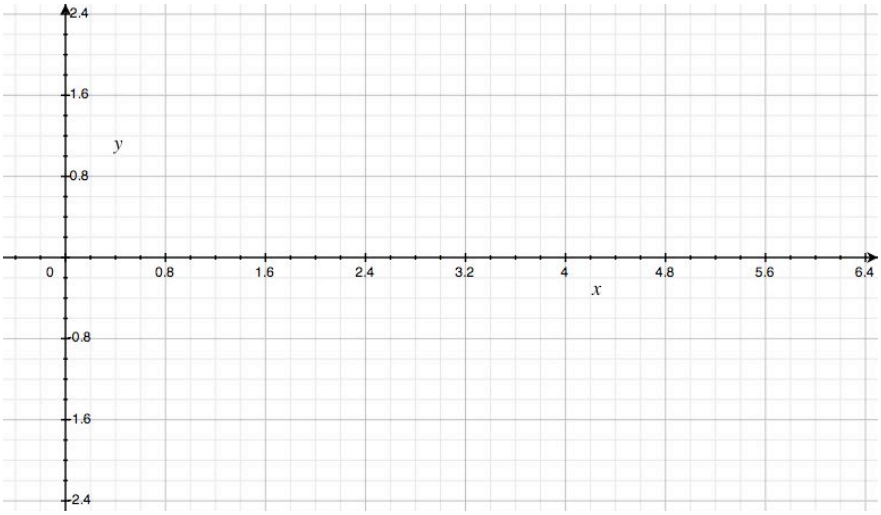
8. Group 8: (In radians from $x = 0 \rightarrow x = 2\pi$)
   a. $y = \tan(x)$
   b. $y = \tan\left(\frac{1}{2}x\right)$
   c. $y = \tan(2x)$



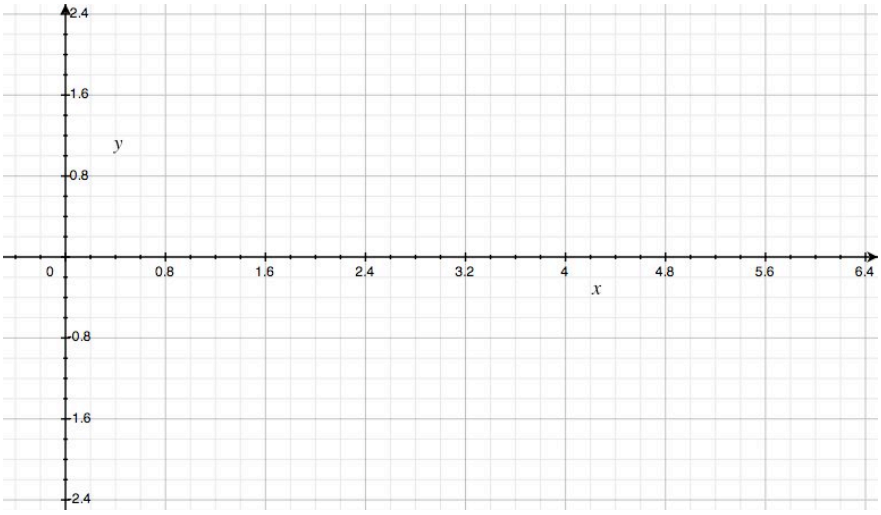
9. Group 9: (In radians from $x = 0 \rightarrow x = 2\pi$)
   a. $y = \sqrt{\tan(x)}$
   b. $y = \sqrt{\frac{1}{2}\tan(x)}$
   c. $y = \sqrt{2\tan(x)}$



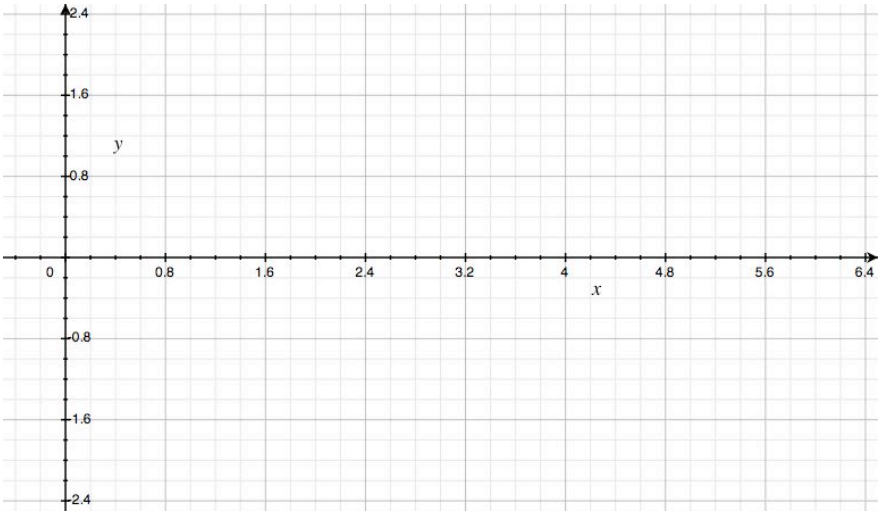
10. Group 10: (In radians from $x = 0 \rightarrow x = 2\pi$)
    a. $y = e^{-\frac{1}{2}x}\cos(2x)$
    b. $y = e^{-x}\cos(2x)$
    c. $y = e^{-2x}\cos(2x)$



11. Group 11: (In radians from $x = 0 \rightarrow x = 2\pi$)
    a. $y = e^{-\frac{1}{2}x}\cos(4x)$
    b. $y = e^{-x}\cos(4x)$
    c. $y = e^{-2x}\cos(4x)$

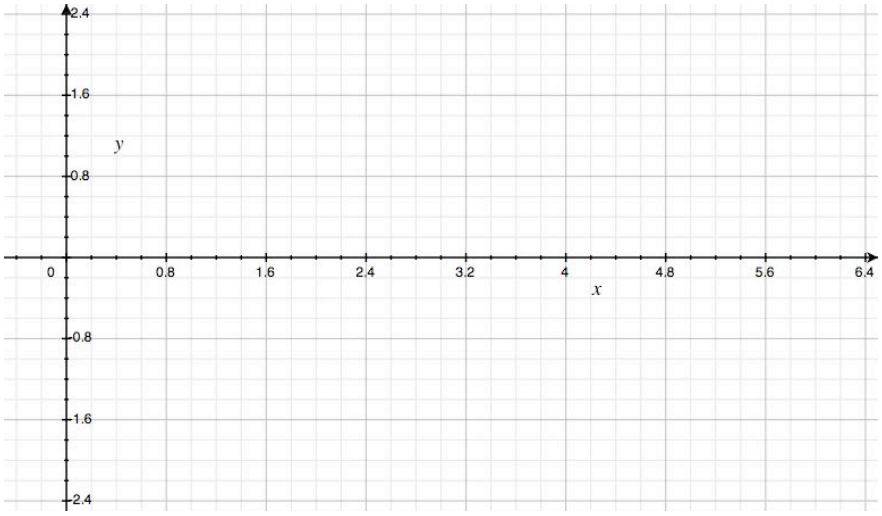


12. Group 12: (In radians from $x = 0 \rightarrow x = 2\pi$)
    a. $y = e^{-\frac{1}{2}x}\cos(8x)$
    b. $y = e^{-x}\cos(8x)$
    c. $y = e^{-2x}\cos(8x)$



13. Miscellaneous Graph(s): (In radians from $x = 0 \rightarrow x = 2\pi$)

a. $y = 2 \sin x \cos x$

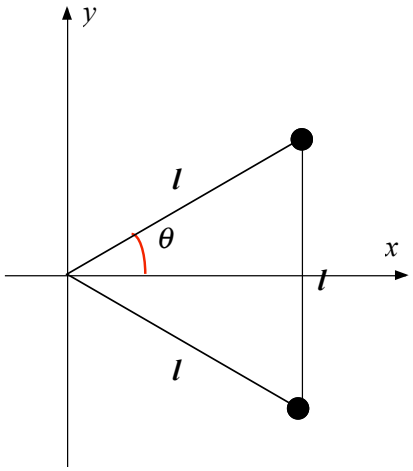


Geometry/Trigonometry in 2-Dimensional and 3-Dimenstional Space

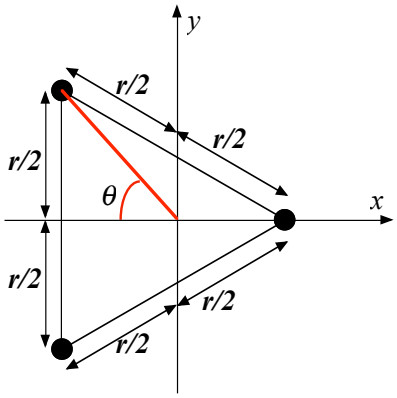
XIII. 2D and 3D space

1. Calculate the exact **distance** for **all of the black dots** in terms of the side lengths, *l* or *r*, and **angle** for **all of the black dots** in degrees, from the origin (0, 0) for 2D and (0, 0, 0) for †3D. [example: ● $\frac{\sqrt{2}}{2}l$ @ 45°]

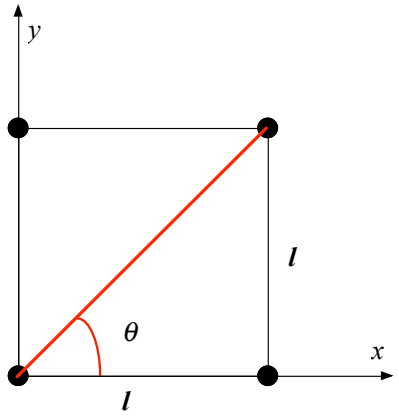
   a. 2 particles [Please note: Some angles are not shown.]



   b. 3 particles [Please note: Some angles are not shown.]



c. 4 particles (2D) [Please note: Some angles are not shown.]



d. †8 particles (3D) [Please note: Some angles are not shown.]