Physics II: Electricity & Magnetism

Section 21.7

Essential Question(s)

- WHAT PRIOR FOUNDATIONAL MATHEMATICS' SKILLS ARE NECESSARY IN PHYSICS II?
- HOW DO WE DESCRIBE THE NATURE OF ELECTROSTATICS AND APPLY IT TO VARIOUS SITUATIONS?
  - *How do we apply integration and the Principle of Superposition to uniformly charged objects?*
  - *How do we identify and apply the fields of highly symmetric charge distributions?*
  - *How do we describe and apply the electric field created by uniformly charged objects?*

Vocabulary

- Static Electricity
- Electric Charge
- Positive / Negative
- Attraction / Repulsion
- Charging / Discharging
- Friction
- Induction
- Conduction
- Law of Conservation of Electric Charge
- Non-polar Molecules
- Polar Molecules
- Ion
- Ionic Compounds
- Force
- Derivative
- Integration (Integrals)
- Test Charge
- Electric Field
- Field Lines
- Electric Dipole
- Dipole Moment

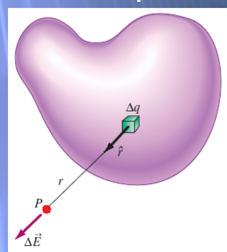
21-7 The Field of a Continuous Distribution

To find the field of a continuous distribution of charge, treat it as a collection of near-point charges:

$$\Delta\vec{E} = \frac{\Delta q}{4\pi\varepsilon_0 r^2}\hat{r}$$

Summing over the infinitesimal fields:

$$\vec{E} = \sum \Delta\vec{E}$$



How do we apply integration and the Principle of Superposition to uniformly charged objects?
How do we identify and apply the fields of highly symmetric charge distributions?
How do we describe and apply the electric field created by uniformly charged objects?

21-7 The Field of a Continuous Distribution

Finally, making the charges infinitesimally small and integrating rather than summing:

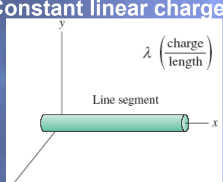
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \hat{r}\frac{dq}{r^2}$$

How do we apply integration and the Principle of Superposition to uniformly charged objects?
How do we identify and apply the fields of highly symmetric charge distributions?
How do we describe and apply the electric field created by uniformly charged objects?

21-7 The Field of a Continuous Distribution

Some types of charge distribution are relatively simple.

Constant linear charge density $\lambda \equiv \frac{Q}{L}$:



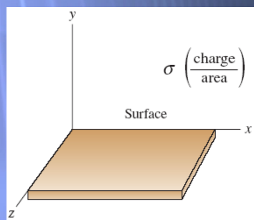
$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0}\int \hat{r}\frac{dx}{r^2}$$

How do we apply integration and the Principle of Superposition to uniformly charged objects?
How do we identify and apply the fields of highly symmetric charge distributions?
How do we describe and apply the electric field created by uniformly charged objects?

21-7 The Field of a Continuous Distribution

Constant surface charge density $\sigma \equiv \frac{Q}{A}$:

$$\vec{E} = \frac{\sigma}{4\pi\varepsilon_0}\int_{\text{surface}} \hat{r}\frac{dS}{r^2}$$

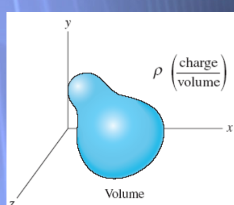


How do we apply integration and the Principle of Superposition to uniformly charged objects?
How do we identify and apply the fields of highly symmetric charge distributions?
How do we describe and apply the electric field created by uniformly charged objects?

21-7 The Field of a Continuous Distribution

Constant volume charge density $\rho \equiv \frac{Q}{V}$:

$$\vec{E} = \frac{\rho}{4\pi\varepsilon_0}\int_{\text{volume}} \hat{r}\frac{dV}{r^2}$$



How do we apply integration and the Principle of Superposition to uniformly charged objects?
How do we identify and apply the fields of highly symmetric charge distributions?
How do we describe and apply the electric field created by uniformly charged objects?

Steps to Determine the E-Field Created by a Uniform Charge Distributions

Graphical:

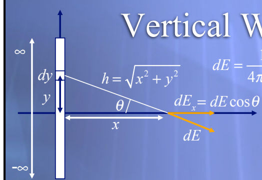
1. Draw a picture of the object and 3-D plane.
2. Label the partial length, area, or volume that is creating the partial E-field.
3. Determine the distance from the charged object to the location of the desired E-Field and label all components and lengths.

Mathematical:

4. Write the formulas for *dE* and the component of the E-field that contributes to the net E-field (i.e. does not cancel due to symmetry).
5. Write the total charge density and solve it for Q.
6. Write the charge density in relation to the partial charge and solve it for the partial charge (*dq*).
7. Set up the integral by determining what key component(s) change.
8. †Solve the integral and write the answer in a concise manner.

†See the instructor, AP Calculus BC students, or Schaum's Mathematical Handbook.

Uniformly Charged Vertical Wire ($^{-}\infty \rightarrow {}^{+}\infty$)



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{h^2};\; dE_x = dE\cos\theta = dE\frac{x}{h} = \frac{1}{4\pi\varepsilon_0}\frac{x}{h^3}dq$$

$$\lambda = \frac{Q}{l} = \frac{Q}{y} \qquad \Rightarrow Q = \lambda y$$

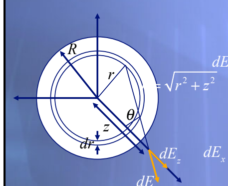
$$\lambda = \frac{dq}{dl} = \frac{dq}{dy} \qquad \Rightarrow dq = \lambda dy$$

$$E_x = \int_0^{E_{tot}} dE_x = \frac{1}{4\pi\varepsilon_0}\int_0^{Q_{tot}} \frac{x}{h^3}\underbrace{dq}_{=\lambda dy} = \frac{1}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{x}{\underbrace{h}_{\sqrt{x^2+y^2}}{}^3}\lambda\, dy = \frac{1}{4\pi\varepsilon_0}\lambda x\int_{-\infty}^{\infty}\frac{1}{\left(x^2+y^2\right)^{3/2}}dy$$

$$E_x = \frac{1}{4\pi\varepsilon_0}\lambda x\left[\frac{y}{x^2\sqrt{x^2+y^2}}\right]_{-\infty}^{\infty} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{x}\left[\frac{y}{\sqrt{x^2+y^2}}\right]_{-\infty}^{\infty} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{x}\left[1-(-1)\right] = \boxed{\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x}}$$

$$E_x = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x}\frac{y}{y} = \frac{1}{2\pi\varepsilon_0}\frac{Q}{xy}$$

Uniformly Charged Disk ($0 \rightarrow R$)



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{h^2};\; dE_z = dE\cos\theta = dE\frac{z}{h} = \frac{1}{4\pi\varepsilon_0}\frac{z}{h^3}dq$$

$$\sigma = \frac{Q}{A} = \frac{Q}{\pi r^2} \qquad \Rightarrow Q = \pi r^2\sigma$$

$$\frac{dA}{dr} = 2\pi r \qquad \Rightarrow dA = 2\pi r dr$$

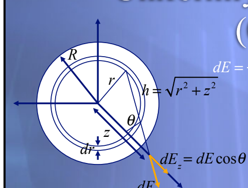
$$\sigma = \frac{dq}{dA} = \frac{dq}{2\pi r dr} \qquad \Rightarrow dq = 2\pi r\sigma dr$$

$$E_z = \int_0^{E_{tot}} dE_z = \frac{1}{4\pi\varepsilon_0}\int_0^{Q_{tot}}\frac{z}{h^3}\underbrace{dq}_{=2\pi r\sigma dr} = \frac{1}{4\pi\varepsilon_0}\int_0^{R}\frac{z}{\underbrace{h}_{\sqrt{z^2+r^2}}{}^3}2\pi r\sigma\, dr$$

$$E_z = \frac{1}{4\pi\varepsilon_0}2\pi\sigma z\int_0^{R}\frac{r}{\left(z^2+r^2\right)^{3/2}}dr = \frac{1}{2\varepsilon_0}\sigma z\left[-\frac{1}{\sqrt{z^2+r^2}}\right]_0^{R} = \frac{1}{2\varepsilon_0}\sigma z\left[\frac{1}{\sqrt{z^2+r^2}}\right]_R^{0}$$

$$E_z = \frac{1}{2\varepsilon_0}\sigma z\left[\frac{1}{z}-\frac{1}{\sqrt{z^2+R^2}}\right] = \frac{\sigma}{2\varepsilon_0}\left[\frac{z}{z}-\frac{z}{\sqrt{z^2+R^2}}\right] = \boxed{\frac{\sigma}{2\varepsilon_0}\left[1-\frac{z}{\sqrt{z^2+R^2}}\right]}$$

Uniformly Charged Disk ($0 \rightarrow \infty$)



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{h^2};\; dE_z = dE\cos\theta = dE\frac{z}{h} = \frac{1}{4\pi\varepsilon_0}\frac{z}{h^3}dq$$

$$\sigma = \frac{Q}{A} = \frac{Q}{\pi r^2} \qquad \Rightarrow Q = \pi r^2\sigma$$

$$\frac{dA}{dr} = 2\pi r \qquad \Rightarrow dA = 2\pi r dr$$

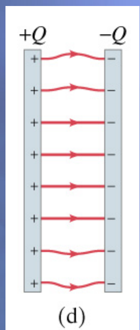
$$\sigma = \frac{dq}{dA} = \frac{dq}{2\pi r dr} \qquad \Rightarrow dq = 2\pi r\sigma dr$$

$$E_z = \int_0^{E_{tot}} dE_z = \frac{1}{4\pi\varepsilon_0}\int_0^{Q_{tot}}\frac{z}{h^3}\underbrace{dq}_{=2\pi r\sigma dr} = \frac{1}{4\pi\varepsilon_0}\int_0^{\infty}\frac{z}{\underbrace{h}_{\sqrt{z^2+r^2}}{}^3}2\pi r\sigma\, dr$$

$$E_z = \frac{1}{4\pi\varepsilon_0}2\pi\sigma z\int_0^{\infty}\frac{r}{\left(z^2+r^2\right)^{3/2}}dr = \frac{1}{2\varepsilon_0}\sigma z\left[-\frac{1}{\sqrt{z^2+r^2}}\right]_0^{\infty} = \frac{1}{2\varepsilon_0}\sigma z\left[\frac{1}{\sqrt{z^2+r^2}}\right]_{\infty}^{0}$$

$$E_z = \frac{1}{2\varepsilon_0}\sigma z\left[\frac{1}{z}-\frac{1}{\sqrt{z^2+\infty^2}}\right] = \frac{\sigma}{2\varepsilon_0}\left[\frac{z}{z}\right] = \boxed{\frac{\sigma}{2\varepsilon_0}}$$

21.8 Field Lines

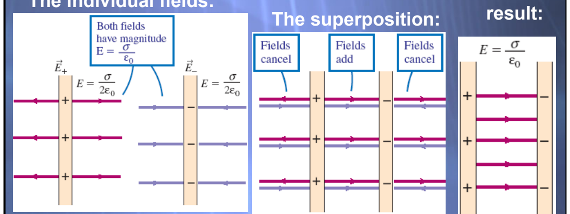


The electric field between two closely spaced, oppositely charged parallel plates is constant.

21-7 The Field of a Continuous Distribution

From the electric field due to a uniform sheet of charge, we can calculate what would happen if we put two oppositely-charged sheets next to each other:

The individual fields: **The superposition:** **The result:**

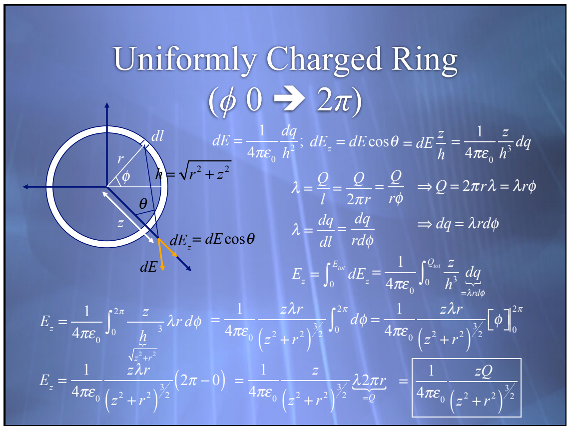


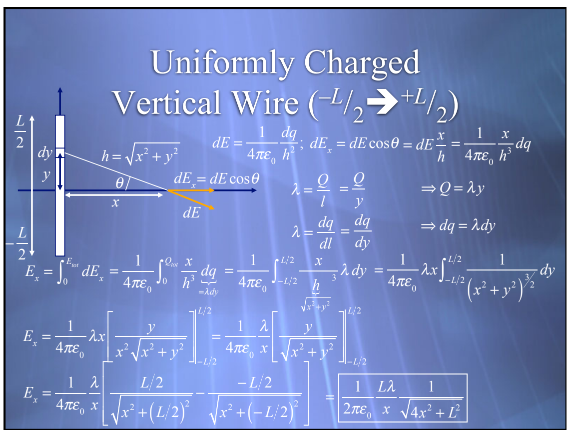
How do we apply integration and the Principle of Superposition to uniformly charged objects?
How do we identify and apply the fields of highly symmetric charge distributions?
How do we describe and apply the electric field created by uniformly charged objects?

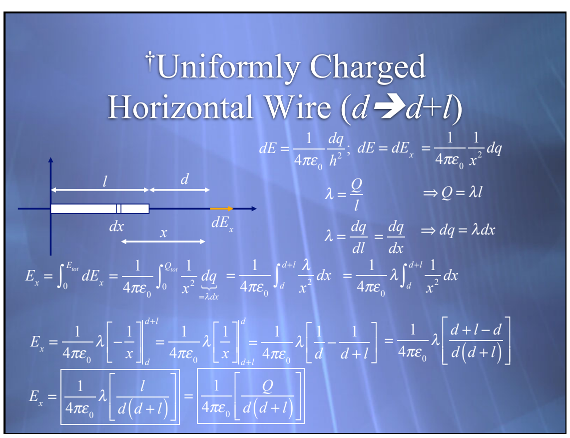
Summary

- How does the result of the Uniformly Charged Disk (0?∞) & the Uniformly Charged Infinite Plate compare? Why is this the case when one is circular and the other is a rectangle?
- HW (Place in your agenda):
  - "Foundational Mathematics' Skills of Physics" Packet (Page 8)
  - For each of the following, complete 1 derivation with reasons for each mathematical step and 3 additional derivations: (*Refer to rubric)
    - *Uniformly Charged Disk (0?∞)
    - *Uniformly Charged Hoop (R_1?R_2) - Deleted for S2009
    - Uniformly Charged Infinite Plate
  - Web Assign 21.8 - 21.11

How do we apply integration and the Principle of Superposition to uniformly charged objects?
How do we identify and apply the fields of highly symmetric charge distributions?
How do we describe and apply the electric field created by uniformly charged objects?


Uniformly Charged Ring
(ϕ 0 ➔ 2π)
dl
r
ϕ
$h=\sqrt{r^2+z^2}$
θ
z
$dE_z = dE\cos\theta$
dE
$dE=\frac{1}{4\pi\varepsilon_0}\frac{dq}{h^2};\ dE_z = dE\cos\theta = dE\frac{z}{h}=\frac{1}{4\pi\varepsilon_0}\frac{z}{h^3}dq$
$\lambda=\frac{Q}{l}=\frac{Q}{2\pi r}=\frac{Q}{r\phi}\quad\Rightarrow Q=2\pi r\lambda=\lambda r\phi$
$\lambda=\frac{dq}{dl}=\frac{dq}{rd\phi}\quad\Rightarrow dq=\lambda rd\phi$
$E_z=\int_0^{E_{tot}}dE_z=\frac{1}{4\pi\varepsilon_0}\int_0^{Q_{tot}}\frac{z}{h^3}\underbrace{dq}_{=\lambda rd\phi}$
$E_z=\frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{z}{\underbrace{h}_{\sqrt{z^2+r^2}}{}^3}\lambda r\,d\phi=\frac{1}{4\pi\varepsilon_0}\frac{z\lambda r}{(z^2+r^2)^{3/2}}\int_0^{2\pi}d\phi=\frac{1}{4\pi\varepsilon_0}\frac{z\lambda r}{(z^2+r^2)^{3/2}}[\phi]_0^{2\pi}$
$E_z=\frac{1}{4\pi\varepsilon_0}\frac{z\lambda r}{(z^2+r^2)^{3/2}}(2\pi-0)=\frac{1}{4\pi\varepsilon_0}\frac{z}{(z^2+r^2)^{3/2}}\underbrace{\lambda 2\pi r}_{=Q}=\boxed{\frac{1}{4\pi\varepsilon_0}\frac{zQ}{(z^2+r^2)^{3/2}}}$


Uniformly Charged
Vertical Wire ($^{-L}/_2$➔$^{+L}/_2$)
$\frac{L}{2}$
dy
y
$h=\sqrt{x^2+y^2}$
θ
x
$dE_x = dE\cos\theta$
dE
$-\frac{L}{2}$
$dE=\frac{1}{4\pi\varepsilon_0}\frac{dq}{h^2};\ dE_x = dE\cos\theta = dE\frac{x}{h}=\frac{1}{4\pi\varepsilon_0}\frac{x}{h^3}dq$
$\lambda=\frac{Q}{l}=\frac{Q}{y}\quad\Rightarrow Q=\lambda y$
$\lambda=\frac{dq}{dl}=\frac{dq}{dy}\quad\Rightarrow dq=\lambda dy$
$E_x=\int_0^{E_{tot}}dE_x=\frac{1}{4\pi\varepsilon_0}\int_0^{Q_{tot}}\frac{x}{h^3}\underbrace{dq}_{=\lambda dy}=\frac{1}{4\pi\varepsilon_0}\int_{-L/2}^{L/2}\frac{x}{\underbrace{h}_{\sqrt{x^2+y^2}}{}^3}\lambda\,dy=\frac{1}{4\pi\varepsilon_0}\lambda x\int_{-L/2}^{L/2}\frac{1}{(x^2+y^2)^{3/2}}dy$
$E_x=\frac{1}{4\pi\varepsilon_0}\lambda x\left[\frac{y}{x^2\sqrt{x^2+y^2}}\right]_{-L/2}^{L/2}=\frac{1}{4\pi\varepsilon_0}\frac{\lambda}{x}\left[\frac{y}{\sqrt{x^2+y^2}}\right]_{-L/2}^{L/2}$
$E_x=\frac{1}{4\pi\varepsilon_0}\frac{\lambda}{x}\left[\frac{L/2}{\sqrt{x^2+(L/2)^2}}-\frac{-L/2}{\sqrt{x^2+(-L/2)^2}}\right]=\boxed{\frac{1}{2\pi\varepsilon_0}\frac{L\lambda}{x}\frac{1}{\sqrt{4x^2+L^2}}}$


†Uniformly Charged
Horizontal Wire (d➔$d+l$)
l
d
dx
x
dE_x
$dE=\frac{1}{4\pi\varepsilon_0}\frac{dq}{h^2};\ dE=dE_x=\frac{1}{4\pi\varepsilon_0}\frac{1}{x^2}dq$
$\lambda=\frac{Q}{l}\quad\Rightarrow Q=\lambda l$
$\lambda=\frac{dq}{dl}=\frac{dq}{dx}\quad\Rightarrow dq=\lambda dx$
$E_x=\int_0^{E_{tot}}dE_x=\frac{1}{4\pi\varepsilon_0}\int_0^{Q_{tot}}\frac{1}{x^2}\underbrace{dq}_{=\lambda dx}=\frac{1}{4\pi\varepsilon_0}\int_d^{d+l}\frac{\lambda}{x^2}dx=\frac{1}{4\pi\varepsilon_0}\lambda\int_d^{d+l}\frac{1}{x^2}dx$
$E_x=\frac{1}{4\pi\varepsilon_0}\lambda\left[-\frac{1}{x}\right]_d^{d+l}=\frac{1}{4\pi\varepsilon_0}\lambda\left[\frac{1}{x}\right]_{d+l}^{d}=\frac{1}{4\pi\varepsilon_0}\lambda\left[\frac{1}{d}-\frac{1}{d+l}\right]=\frac{1}{4\pi\varepsilon_0}\lambda\left[\frac{d+l-d}{d(d+l)}\right]$
$E_x=\boxed{\frac{1}{4\pi\varepsilon_0}\lambda\left[\frac{l}{d(d+l)}\right]}=\boxed{\frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{d(d+l)}\right]}$