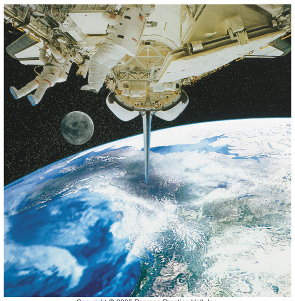
Chapter 5

Circular Motion; Gravitation



Units of Chapter 5

- Kinematics of Uniform Circular Motion
- Dynamics of Uniform Circular Motion
- Highway Curves, Banked and Unbanked
- Nonuniform Circular Motion
- Centrifugation
- Newton's Law of Universal Gravitation

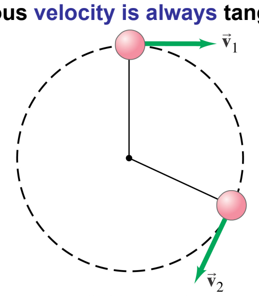
Units of Chapter 5

- Gravity Near the Earth's Surface; Geophysical Applications
- Satellites and "Weightlessness"
- Kepler's Laws and Newton's Synthesis
- Types of Forces in Nature

5-1 Kinematics of Uniform Circular Motion

Uniform circular motion: motion in a **circle** of constant **radius** at constant **speed**

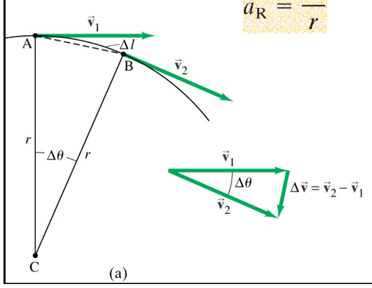
Instantaneous velocity is always **tangent** to circle.

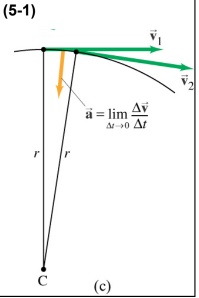


5-1 Kinematics of Uniform Circular Motion

Looking at the change in velocity in the limit that the time interval becomes infinitesimally small, we see that

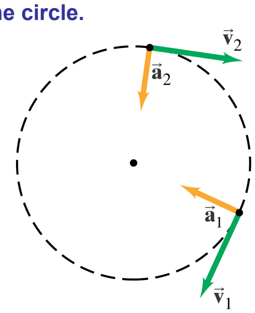
$$a_R = \frac{v^2}{r} \quad (5\text{-}1)$$





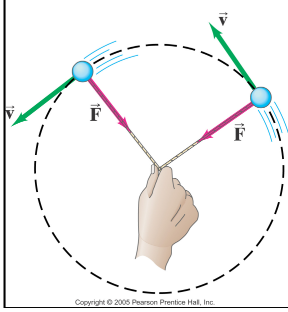
5-1 Kinematics of Uniform Circular Motion

This acceleration is called the **centripetal**, or radial, acceleration, and it points towards the center of the circle.



5-2 Dynamics of Uniform Circular Motion

For an object to be in uniform circular motion, there must be a **net force** acting on it.

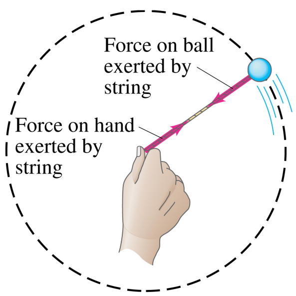


We already know the acceleration, so can immediately write the force:

$$\Sigma F_R = ma_R = m\frac{v^2}{r} \tag{5-1}$$

5-2 Dynamics of Uniform Circular Motion

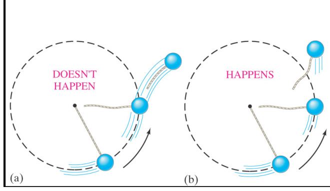
We can see that the force must be **inward** by thinking about a ball on a string:



5-2 Dynamics of Uniform Circular Motion

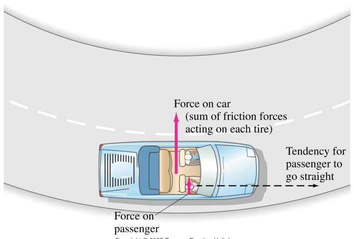
There is no **centrifugal** force pointing outward; what happens is that the natural tendency of the object to move in a straight line must be overcome.

If the centripetal force vanishes, the object flies off **tangent** to the circle.



5-3 Highway Curves, Banked and Unbanked

When a car goes around a **curve**, there must be a net force towards the center of the circle of which the curve is an arc. If the road is flat, that force is supplied by **friction**.



5-3 Highway Curves, Banked and Unbanked

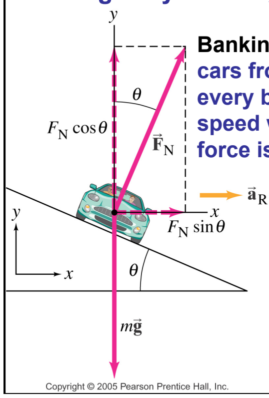
If the frictional force is **insufficient**, the car will tend to move more nearly in a **straight line**, as the skid marks show.

5-3 Highway Curves, Banked and Unbanked

As long as the tires do not slip, the friction is **static**. If the tires do start to slip, the friction is **kinetic**, which is bad in two ways:

1. The kinetic frictional force is **smaller** than the static.
2. The static frictional force can point towards the center of the circle, but the kinetic frictional force **opposes** the direction of motion, making it very difficult to regain control of the car and continue around the curve.

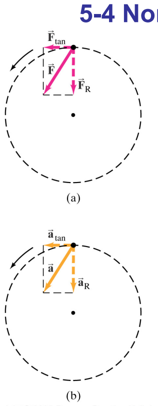
5-3 Highway Curves, Banked and Unbanked



Banking the curve can help keep cars from skidding. In fact, for every banked curve, there is one speed where the entire centripetal force is supplied by the horizontal component of the **normal** force, and no friction is required. This occurs when:

$$F_N \sin\theta = m\frac{v^2}{r}$$

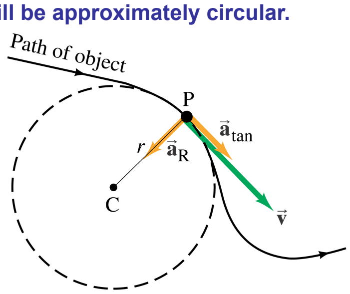
5-4 Nonuniform Circular Motion



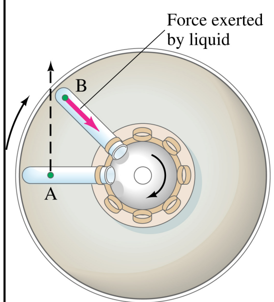
If an object is moving in a circular path but at varying **speeds**, it must have a **tangential** component to its acceleration as well as the **radial** one.

5-4 Nonuniform Circular Motion

This concept can be used for an object moving along any **curved path**, as a small segment of the path will be approximately circular.



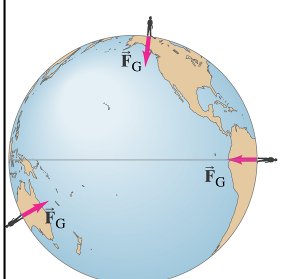
5-5 Centrifugation



A **centrifuge** works by spinning very fast. This means there must be a very large **centripetal** force. The object at A would go in a straight line but for this force; as it is, it winds up at B.

5-6 Newton's Law of Universal Gravitation

If the force of gravity is being exerted on objects on Earth, what is the **origin** of that force?



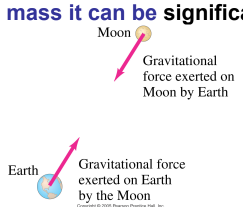
Newton's realization was that the force must come from the **Earth**.

He further realized that this force must be what keeps the **Moon** in its orbit.

5-6 Newton's Law of Universal Gravitation

The gravitational force on you is one-half of a Third Law pair: the **Earth** exerts a **downward** force on you, and you exert an **upward** force on the Earth.

When there is such a **disparity** in masses, the reaction force is undetectable, but for bodies more equal in mass it can be **significant**.



5-6 Newton's Law of Universal Gravitation

Therefore, the gravitational force must be proportional to **both** masses.

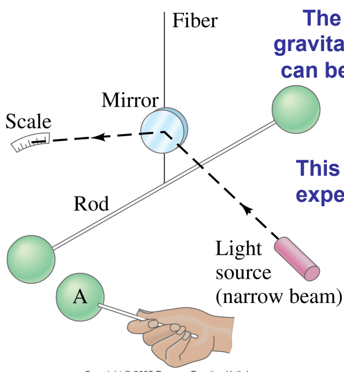
By observing planetary orbits, Newton also concluded that the gravitational force must decrease as the **inverse** of the **square** of the distance between the masses.

In its final form, the Law of Universal Gravitation reads:

$$F = G\frac{m_1 m_2}{r^2} \quad (5\text{-}4)$$

where $G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$

5-6 Newton's Law of Universal Gravitation



The **magnitude** of the gravitational constant *G* can be measured in the laboratory.

This is the **Cavendish** experiment.

5-7 Gravity Near the Earth's Surface; Geophysical Applications

Now we can relate the **gravitational constant** to the **local acceleration** of gravity. We know that, on the surface of the Earth:

$$mg = G\frac{m m_E}{r_E^2}$$

Solving for *g* gives:

$$g = G\frac{m_E}{r_E^2} \quad (5\text{-}5)$$

Now, knowing *g* and the radius of the Earth, the mass of the Earth can be calculated:

$$m_E = \frac{g r_E^2}{G} = \frac{(9.80\ \mathrm{m/s^2})(6.38 \times 10^6\ \mathrm{m})^2}{6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}} = 5.98 \times 10^{24}$$

5-7 Gravity Near the Earth's Surface; Geophysical Applications

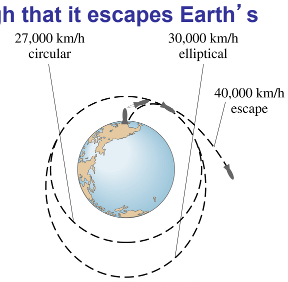
TABLE 5–1
Acceleration Due to Gravity at Various Locations on Earth

Location	Elevation (m)	g (m/s^2)
New York	0	9.803
San Francisco	0	9.800
Denver	1650	9.796
Pikes Peak	4300	9.789
Sydney, Australia	0	9.798
Equator	0	9.780
North Pole (calculated)	0	9.832

The acceleration due to gravity **varies** over the Earth's surface due to altitude, local geology, and the shape of the Earth, which is not quite spherical.

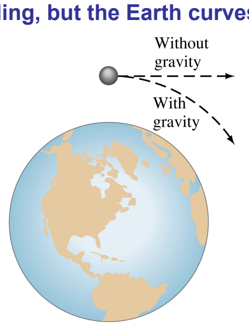
5-8 Satellites and "Weightlessness"

Satellites are routinely put into orbit around the Earth. The **tangential speed** must be high enough so that the satellite does not return to Earth, but not so high that it escapes Earth's gravity altogether.



5-8 Satellites and "Weightlessness"

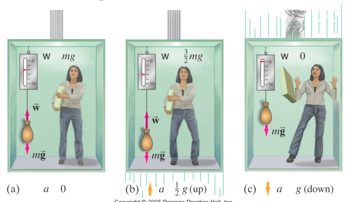
The satellite is kept in **orbit** by its **speed** – it is continually falling, but the Earth curves from underneath it.



5-8 Satellites and "Weightlessness"

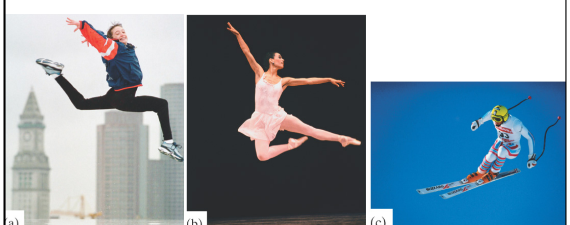
Objects in orbit are said to experience **weightlessness**. They do have a gravitational force acting on them, though!

The satellite and all its contents are in **free fall**, so there is no **normal force**. This is what leads to the experience of weightlessness.



5-8 Satellites and "Weightlessness"

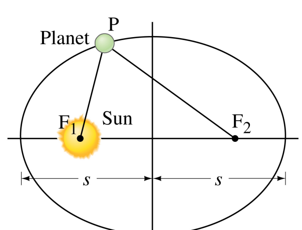
More properly, this effect is called **apparent weightlessness**, because the gravitational force still exists. It can be experienced on Earth as well, but only briefly:



5-9 Kepler's Laws and Newton's Synthesis

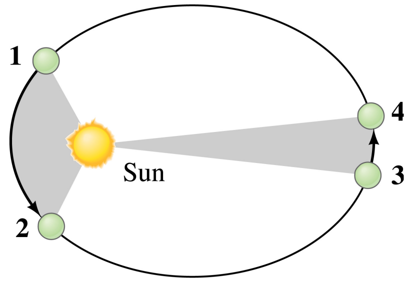
Kepler's laws describe **planetary motion.**

1. The orbit of each planet is an **ellipse**, with the Sun at one focus.



5-9 Kepler's Laws and Newton's Synthesis

2. An imaginary line drawn from each planet to the Sun sweeps out **equal areas** in **equal times**.



5-9 Kepler's Laws and Newton's Synthesis

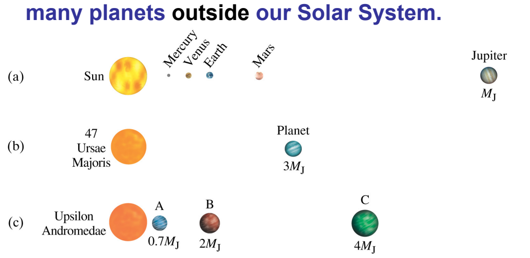
The ratio of the square of a planet's orbital **period** is proportional to the cube of its mean **distance** from the Sun.

TABLE 5–2 Planetary Data Applied to Kepler's Third Law

Planet	Mean Distance from Sun, s (10^6 km)	Period, T (Earth years)	s^3/T^2 (10^{24} km^3/y^2)
Mercury	57.9	0.241	3.34
Venus	108.2	0.615	3.35
Earth	149.6	1.0	3.35
Mars	227.9	1.88	3.35
Jupiter	778.3	11.86	3.35
Saturn	1427	29.5	3.34
Uranus	2870	84.0	3.35
Neptune	4497	165	3.34
Pluto	5900	248	3.34

5-9 Kepler's Laws and Newton's Synthesis

Kepler's laws can be derived from **Newton's laws**. Irregularities in planetary motion led to the discovery of **Neptune**, and irregularities in stellar motion have led to the discovery of many planets **outside** our Solar System.



5-10 Types of Forces in Nature

Modern physics now recognizes four fundamental forces:

1. Gravity
2. Electromagnetism
3. Weak nuclear force (responsible for some types of radioactive decay)
4. Strong nuclear force (binds protons and neutrons together in the nucleus)

5-10 Types of Forces in Nature

So, what about friction, the normal force, tension, and so on?

Except for gravity, the forces we experience every day are due to electromagnetic forces acting at the atomic level.

Summary of Chapter 5

• An object moving in a circle at constant speed is in uniform circular motion.

• It has a centripetal acceleration $a_R = \frac{v^2}{r}$

• There is a centripetal force given by

$$\Sigma F_R = m a_R = m \frac{v^2}{r}$$

•The centripetal force may be provided by friction, gravity, tension, the normal force, or others.

Summary of Chapter 5

- Newton's law of universal gravitation:

$$F = G\frac{m_1 m_2}{r^2}$$

- Satellites are able to stay in Earth orbit because of their large tangential speed.