CHAPTER 11: Vibrations and Waves

Solution Guide to WebAssign Problems

11.1 [1] The particle would travel four times the amplitude: from $x = A$ to $x = 0$ to $x = -A$ to $x = 0$ to $x = A$. So the total distance $= 4A = 4(0.18\ \text{m}) = \boxed{0.72\ \text{m}}$.

11.2 [2] The spring constant is the ratio of applied force to displacement.

$$k = \frac{F}{x} = \frac{180\ \text{N} - 75\ \text{N}}{0.85\ \text{m} - 0.65\ \text{m}} = \frac{105\ \text{N}}{0.20\ \text{m}} = \boxed{5.3 \times 10^2\ \text{N/m}}$$

11.3 [3] The spring constant is found from the ratio of applied force to displacement.

$$k = \frac{F}{x} = \frac{mg}{x} = \frac{(68\ \text{kg})(9.8\ \text{m/s}^2)}{5 \times 10^{-3}\ \text{m}} = 1.333 \times 10^5\ \text{N/m}$$

The frequency of oscillation is found from the total mass and the spring constant.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1.333 \times 10^5\ \text{N/m}}{1568\ \text{kg}}} = 1.467\ \text{Hz} \approx \boxed{1.5\ \text{Hz}}$$

11.4 [7] The relationship between frequency, mass, and spring constant is $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

(*a*) $$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \rightarrow \quad k = 4\pi^2 f^2 m = 4\pi^2 (4.0\ \text{Hz})^2 (2.5 \times 10^{-4}\ \text{kg}) = 0.1579\ \text{N/m} \approx \boxed{0.16\ \text{N/m}}$$

(*b*) $$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{0.1579\ \text{N/m}}{5.0 \times 10^{-4}\ \text{kg}}} = \boxed{2.8\ \text{Hz}}$$

11.5 [9] (*a*) At equilibrium, the velocity is its maximum.

$$v_{\text{max}} = \sqrt{\frac{k}{m}}A = \omega A = 2\pi f A = 2\pi(3\ \text{Hz})(0.13\ \text{m}) = 2.450\ \text{m/s} \approx \boxed{2.5\ \text{m/s}}$$

(*b*) From Equation (11-5), we find the velocity at any position.

$$v = \pm v_{\text{max}}\sqrt{1 - \frac{x^2}{A^2}} = \pm(2.45\ \text{m/s})\sqrt{1 - \frac{(0.10\ \text{m})^2}{(0.13\ \text{m})^2}} = \pm 1.565\ \text{m/s} \approx \boxed{\pm 1.6\ \text{m/s}}$$

(*c*) $$E_{\text{total}} = \tfrac{1}{2} m v_{\text{max}}^2 = \tfrac{1}{2}(0.60\ \text{kg})(2.45\ \text{m/s})^2 = 1.801\ \text{J} \approx \boxed{1.8\ \text{J}}$$

(*d*) Since the object has a maximum displacement at $t = 0$, the position will be described by the cosine function.

$$x = (0.13\text{ m})\cos\left(2\pi(3.0\,\text{Hz})t\right) \rightarrow \boxed{x = (0.13\text{ m})\cos(6.0\,\pi t)}$$

11.6 [16] The general form of the motion is $x = A\cos\omega t = 0.45\cos 6.40t$.

(*a*) The amplitude is $A = x_{\text{max}} = \boxed{0.45\text{ m}}$.

(*b*) The frequency is found by $\omega = 2\pi f = 6.40\text{ s}^{-1} \rightarrow f = \dfrac{6.40\text{ s}^{-1}}{2\pi} = 1.019\text{ Hz} \approx \boxed{1.02\text{ Hz}}$

(*c*) The total energy is given by

$$E_{\text{total}} = \tfrac{1}{2}mv_{\text{max}}^2 = \tfrac{1}{2}m(\omega A)^2 = \tfrac{1}{2}(0.60\text{ kg})\left[(6.40\text{ s}^{-1})(0.45\text{ m})\right]^2 = 2.488\text{ J} \approx \boxed{2.5\text{ J}}$$

(*d*) The potential energy is given by

$$E_{\text{potential}} = \tfrac{1}{2}kx^2 = \tfrac{1}{2}m\omega^2x^2 = \tfrac{1}{2}(0.60\text{ kg})(6.40\text{ s}^{-1})^2(0.30\text{ m})^2 = 1.111\text{J} \approx \boxed{1.1\text{ J}}$$

The kinetic energy is given by

$$E_{\text{kinetic}} = E_{\text{total}} - E_{\text{potential}} = 2.488\text{ J} - 1.111\text{ J} = 1.377\text{ J} \approx \boxed{1.4\text{ J}}$$

11.7 [19] (*a*) The general equation for SHM is Equation (11-8c), $y = A\cos(2\pi t/T)$. For the pumpkin,

$$\boxed{y = (0.18\text{ m})\cos\left(\frac{2\pi t}{0.65\text{ s}}\right)}.$$

(*b*) The time to return back to the equilibrium position is one-quarter of a period.

$$t = \tfrac{1}{4}T = \tfrac{1}{4}(0.65\text{ s}) = \boxed{0.16\text{ s}}$$

(*c*) The maximum speed is given by the angular frequency times the amplitude.

$$v_{\text{max}} = \omega A = \frac{2\pi}{T}A = \frac{2\pi}{0.65\text{ s}}(0.18\text{ m}) = \boxed{1.7\text{ m/s}}$$

(*d*) The maximum acceleration is given by

$$a_{\text{max}} = \omega^2 A = \left(\frac{2\pi}{T}\right)^2 A = \frac{4\pi^2}{(0.65\text{ s})^2}(0.18\text{ m}) = \boxed{17\text{ m/s}^2}.$$

The maximum acceleration is first attained at the release point of the pumpkin.

11.8 [22] (*a*) For A, the amplitude is $A_A = \boxed{2.5\text{ m}}$. For B, the amplitude is $A_B = \boxed{3.5\text{ m}}$.

(*b*) For A, the frequency is 1 cycle every 4.0 seconds, so $f_A = \boxed{0.25\text{ Hz}}$. For B, the frequency is 1 cycle every 2.0 seconds, so $f_B = \boxed{0.50\text{ Hz}}$.

(*c*) For C, the period is $T_A = \boxed{4.0\text{ s}}$. For B, the period is $T_B = \boxed{2.0\text{ s}}$

(*d*) Object A has a displacement of 0 when $t = 0$, so it is a sine function.

$$x_A = A_A \sin(2\pi f_A t) \rightarrow \boxed{x_A = \boxed{2.5\text{ m}} \sin\left(\frac{\pi}{2}t\right)}$$

Object B has a maximum displacement when $t = 0$, so it is a cosine function.

$$x_B = A_B \cos(2\pi f_B t) \rightarrow \boxed{x_B = (3.5\text{ m})\cos(\pi t)}$$

11.9 [23] (*a*) Find the period and frequency from the mass and the spring constant.

$$T = 2\pi\sqrt{m/k} = 2\pi\sqrt{0.755\text{ kg}/(124\text{ N/m})} = \boxed{0.490\text{ s}} \qquad f = 1/T = 1/(0.490\text{ s}) = \boxed{2.04\text{ Hz}}$$

(*b*) The initial speed is the maximum speed, and that can be used to find the amplitude.

$$V_{max} = A\sqrt{k/m} \rightarrow A = v_{max}\sqrt{m/k} = (2.96\text{ m/s})\sqrt{0.755\text{ kg}/(124\text{ N/m})} = \boxed{0.231\text{ m}}$$

(*c*) The maximum acceleration can be found from the mass, spring constant, and amplitude

$$a_{max} = A\,k/m = (0.231\text{ m})(124\text{ N/m})/(0.755\text{ kg}) = \boxed{37.9\text{ m/s}^2}$$

(*d*) Because the mass started at the equilibrium position of $x = 0$, the position function will be proportional to the sine function.

$$x = (0.231\text{ m})\sin[2\pi(2.04\text{ Hz})t] \rightarrow \boxed{x = (0.231\text{ m})\sin(4.08\pi t)}$$

(*e*) The maximum energy is the kinetic energy that the object has when at the equilibrium position.

$$E = \tfrac{1}{2}mv_{max}^2 = \tfrac{1}{2}(0.755\text{ kg})(2.96\text{ m/s})^2 = \boxed{3.31\text{ J}}$$

11.10 [28] (*a*) The period is given by $T = \dfrac{60\text{ s}}{36\text{ cycles}} = \boxed{1.7\text{ s/cycle}}$.

(*b*) The frequency is given by $f = \dfrac{36 \text{ cycles}}{60 \text{ s}} = \boxed{0.60 \text{ Hz}}$.

11.11 [29] The period of a pendulum is given by $T = 2\pi\sqrt{L/g}$. Solve for the length using a period of 2.0 seconds.

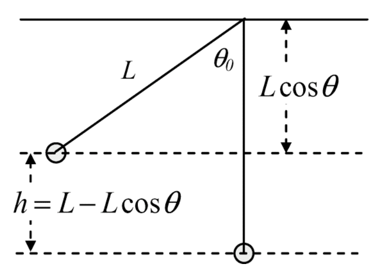
$$T = 2\pi\sqrt{L/g} \quad \rightarrow \quad L = \frac{T^2 g}{4\pi^2} = \frac{(2.0 \text{ s})^2 (9.8 \text{ m/s}^2)}{4\pi^2} = \boxed{0.99 \text{ m}}$$

11.12 [30] The period of a pendulum is given by $T = 2\pi\sqrt{L/g}$. The length is assumed to be the same for the pendulum both on Mars and on Earth.

$$T = 2\pi\sqrt{L/g} \quad \rightarrow \quad \frac{T_{\text{Mars}}}{T_{\text{Earth}}} = \frac{2\pi\sqrt{L/g_{\text{Mars}}}}{2\pi\sqrt{L/g_{\text{Earth}}}} = \sqrt{\frac{g_{\text{Earth}}}{g_{\text{Mars}}}} \quad \rightarrow$$

$$T_{\text{Mars}} = T_{\text{Earth}}\sqrt{\frac{g_{\text{Earth}}}{g_{\text{Mars}}}} = (0.80 \text{ s})\sqrt{\frac{1}{0.37}} = \boxed{1.3 \text{ s}}$$

11.13 [32] (*a*) The frequency can be found from the length of the pendulum, and the acceleration due to gravity.



$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.80 \text{ m/s}^2}{0.760 \text{ m}}} = 0.57151 \text{ Hz} \approx \boxed{0.572 \text{ Hz}}$$

(*b*) To find the speed at the lowest point, use the conservation of energy relating the lowest point to the release point of the pendulum. Take the lowest point to be the zero level of gravitational potential energy.

$$E_{\text{top}} = E_{\text{bottom}} \quad \rightarrow \quad KE_{\text{top}} + PE_{\text{top}} = KE_{\text{bottom}} + PE_{\text{bottom}}$$

$$0 + mg\left(L - L \cos \theta_{\text{o}}\right) = \tfrac{1}{2} m v_{\text{bottom}}^2 + 0$$

$$v_{\text{bottom}} = \sqrt{2gL(1 - \cos\theta_o)}\sqrt{2\left(9.80 \text{ m/s}^2\right)(0.760 \text{ m})(1 - \cos 12.0^\circ)} = \boxed{0.571 \text{ m/s}}$$

(*c*) The total energy can be found from the kinetic energy at the bottom of the motion.

$$E_{\text{total}} = \tfrac{1}{2} m v_{\text{bottom}}^2 = \tfrac{1}{2}(0.365 \text{ kg})(0.571 \text{ m/s})^2 = \boxed{5.95 \times 10^{-2} \text{ J}}$$

11.14 [36] The wave speed is given by $v = \lambda f$. The period is 3.0 seconds, and the wavelength is 6.5 m.

$$v = \lambda f = \lambda / T = (6.5 \text{ m})/(3.0 \text{ s}) = \boxed{2.2 \text{ m/s}}$$

11.15 [37] The distance between wave crests is the wavelength of the wave.

$\lambda = v/f = 343\,\text{m/s}/262\text{ Hz} = \boxed{1.31\text{ m}}$

11.16 [38] To find the wavelength, use $\lambda = v/f$.

$$\text{AM}: \quad \lambda_1 = \frac{v}{f_1} = \frac{3.00\times10^8\text{ m/s}}{550\times10^3\text{ Hz}} = 545\text{ m} \qquad \lambda_2 = \frac{v}{f_2} = \frac{3.00\times10^8\text{ m/s}}{1600\times10^3\text{ Hz}} = 188\text{ m} \qquad \boxed{\text{AM}: 190\text{ m to }550\text{ m}}$$

$$\text{FM}: \quad \lambda_1 = \frac{v}{f_1} = \frac{3.00\times10^8\text{ m/s}}{88.0\times10^6\text{ Hz}} = 3.41\text{ m} \qquad \lambda_2 = \frac{v}{f_2} = \frac{3.00\times10^8\text{ m/s}}{108\times10^6\text{ Hz}} = 2.78\text{ m} \qquad \boxed{\text{FM}: 2.78\text{ m to }3.41\text{ m}}$$

11.17 [43] The speed of the water wave is given by $v = \sqrt{B/\rho}$, where B is the bulk modulus of water, from Table 9-1, and ρ is the density of sea water, from Table 10-1. The wave travels twice the depth of the ocean during the elapsed time.

$$v = \frac{2L}{t} \quad \rightarrow \quad L = \frac{vt}{2} = \frac{t}{2}\sqrt{\frac{B}{\rho}} = \frac{3.0\text{ s}}{2}\sqrt{\frac{2.0\times10^9\text{ N/m}^2}{1.025\times10^3\text{ kg/m}^3}} = \boxed{2.1\times10^3\text{ m}}$$

11.18 [46] (*a*) Assume that the earthquake waves spread out spherically from the source. Under those conditions, Eq. (11-16b) applies, stating that intensity is inversely proportional to the square of the distance from the source of the wave.

$$I_{20\text{ km}}/I_{10\text{ km}} = (10\text{ km})^2/(20\text{ km})^2 = \boxed{0.25}$$

(*b*) The intensity is proportional to the square of the amplitude, and so the amplitude is inversely proportional to the distance from the source of the wave.

$$A_{20\text{ km}}/A_{10\text{ km}} = 10\text{ km}/20\text{ km} = \boxed{0.50}$$

11.19 [48] From Equation (11-18), if the speed, medium density, and frequency of the two waves are the same, then the intensity is proportional to the square of the amplitude.

$$I_2/I_1 = E_2/E_1 = A_2^2/A_1^2 = 2 \quad \rightarrow \quad A_2/A_1 = \sqrt{2} = \boxed{1.41}$$

The more energetic wave has the larger amplitude.

11.20 [52] The frequencies of the harmonics of a string that is fixed at both ends are given by $f_n = nf_1$, and so the first four harmonics are $\boxed{f_1 = 440\text{ Hz},\ f_2 = 880\text{ Hz},\ f_3 = 1320\text{ Hz},\ f_4 = 1760\text{ Hz}}$.

11.21 [53] The fundamental frequency of the full string is given by $f_{\text{unfingered}} = \frac{v}{2L} = 294\text{ Hz}$. If the length is reduced to 2/3 of its current value, and the velocity of waves on the string is not changed, then the new frequency will be

$$f_{\text{fingered}} = \frac{v}{2\left(\frac{2}{3}L\right)} = \frac{3}{2}\frac{v}{2L} = \left(\frac{3}{2}\right)f_{\text{unfingered}} = \left(\frac{3}{2}\right)294\text{ Hz} = \boxed{441\text{ Hz}}$$

11.22 [56] Since $f_n = nf_1$, two successive overtones differ by the fundamental frequency, as shown below.

$$\Delta f = f_{n+1} - f_n = (n+1)f_1 - nf_1 = f_1 = 350\text{ Hz} - 280\text{ Hz} = \boxed{70\text{ Hz}}$$

11.23 [57] The speed of waves on the string is given by equation (11-13), $v = \sqrt{\frac{F_{\text{T}}}{m/L}}$. The resonant frequencies of a string with both ends fixed are given by equation (11-19b), $f_n = \frac{nv}{2L_{\text{vib}}}$, where L_{vib} is the length of the portion that is actually vibrating. Combining these relationships allows the frequencies to be calculated.

$$f_n = \frac{n}{2L_{\text{vib}}}\sqrt{\frac{F_{\text{T}}}{m/L}} \qquad f_1 = \frac{1}{2(0.62\text{ m})}\sqrt{\frac{520\text{ N}}{(3.6\times10^{-3}\text{ kg})/(0.90\text{ m})}} = 290.77\text{ Hz}$$

$$f_2 = 2f_1 = 581.54\text{ Hz} \qquad f_3 = 3f_1 = 872.31\text{ Hz}$$

So the three frequencies are $\boxed{290\text{ Hz, }580\text{ Hz, }870\text{ Hz}}$, to 2 significant figures.

11.24 [58] From Equation (11-19b), $f_n = \frac{nv}{2L}$, we see that the frequency is proportional to the wave speed on the stretched string. From equation (11-13), $v = \sqrt{\frac{F_{\text{T}}}{m/L}}$, we see that the wave speed is proportional to the square root of the tension. Thus the frequency is proportional to the square root of the tension.

$$\sqrt{\frac{F_{\text{T2}}}{F_{\text{T1}}}} = \frac{f_2}{f_1} \quad \rightarrow \quad F_{\text{T2}} = \left(\frac{f_2}{f_1}\right)^2 F_{\text{T1}} = \left(\frac{200\text{ Hz}}{205\text{ Hz}}\right)^2 F_{\text{T1}} = 0.952\, F_{\text{T1}}$$

Thus the tension should be $\boxed{\text{decreased by }4.8\%}$.

11.25 [62] The speed in the second medium can be found from the law of refraction, Equation (11-20).

$$\frac{\sin\,\theta_2}{\sin\,\theta_1} = \frac{v_2}{v_1} \quad \rightarrow \quad v_2 = v_1\frac{\sin\theta_2}{\sin\theta} = (8.0\text{ km/s})\left(\frac{\sin\,35^\circ}{\sin\,47^\circ}\right) = \boxed{6.3\text{ km/s}}$$

11.26 [63] The angle of refraction can be found from the law of refraction, Equation (11-20).

$$\frac{\sin\ \theta_2}{\sin\ \theta_1}=\frac{v_2}{v_1} \quad \rightarrow \quad \sin\ \theta_2=\sin\theta_1\frac{v_2}{v_1}=\sin\ 34^{\circ}\frac{2.1\ \text{m/s}}{2.8\ \text{m/s}}=0.419 \quad \rightarrow \quad \theta_2=\sin^{-1}0.419=\boxed{25^{\circ}}$$

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\sqrt{\frac{GA/L_o}{\rho V}}=\frac{1}{2\pi}\sqrt{\frac{GA/L_o}{\rho A L_o}}=\frac{1}{2\pi}\sqrt{\frac{G}{\rho L_o^2}}$$

$$=\frac{1}{2\pi}\sqrt{\frac{520\ \text{N/m}^2}{\left(1300\ \text{kg/m}^3\right)\left(4.0\times10^{-2}\ \text{m}\right)^2}}=\boxed{2.5\ \text{Hz}}$$