# Experiment 8: Projectile Velocity—Approximate Method

**EQUIPMENT NEEDED:**

- launcher
- C-clamp (optional)
- string
- Steel ball
- Mass balance

## Purpose:

The muzzle velocity of the projectile launcher is determined by launching the ball into the pendulum and observing the angle to which the pendulum swings.

As derived earlier in this manual, the equation for the velocity of the ball is approximately

$$v_b = \frac{M}{m}\sqrt{2gR_{cm}(1-\cos\theta)}$$

where M is the mass of the pendulum and ball combined, m is the mass of the ball, g is the acceleration of gravity, $R_{cm}$ is the distance from the pivot to the center of mass of the pendulum, and θ is the angle reached by the pendulum.

## Setup:

① Attach the Projectile Launcher to the ballistic pendulum mount at the level of the ball catcher. Make sure that the pendulum can hang vertically without touching the launcher.

② Clamp the pendulum base to the table, if a clamp is available. Make sure that the clamp does not interfere with the pendulum swing. (It is possible to get very good results without clamping to the table, as long as the base is held firmly to the table when the ball is fired.)

## Procedure:

① Latch the pendulum at 90° so it is out of the way, then load the projectile launcher. Allow the pendulum to hang freely, and move the angle indicator to zero degrees.

② Fire the launcher and record the angle reached. If you want to do the experiment with a lower or higher angle, add or remove mass to the pendulum. Repeat these test measurements until you are satisfied with the mass of the pendulum.

③ Once you have chosen the mass to use for your experiment, remove the pendulum from the base by unscrewing and removing the pivot axle. Using the mass balance, find the mass of the pendulum and ball together. Record this value as M in table 8.1.

④ Measure the mass of the ball, and record this as m.

⑤ Tie a loop in the string, and hang the pendulum from the loop. (See figure 8.1) With the ball latched in position in the ball catcher, adjust the position of the pendulum in this loop until it balances. Measure the distance from the pivot point to this balance point, and record it as $R_{cm}$. You may find it easier to do this by balancing the pendulum on the edge of a ruler or similar object.

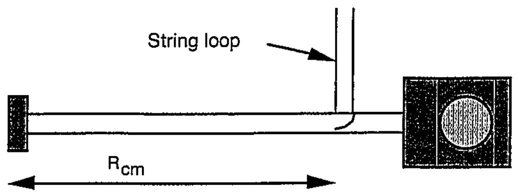


**Figure 8.1**

⑥ Replace the pendulum in the base, making sure that it is facing the right way. Be sure that the angle indicator is to the right of the pendulum rod.

⑦ Load the launcher, then set the angle indicator to an angle 1-2° less than that reached in step 2. This will nearly eliminate the drag on the pendulum caused by the indicator, since the pendulum will only move the indicator for the last few degrees.

Fire the launcher, and record the angle reached by the pendulum in table 8.1. Repeat this several times, setting the angle indicator to a point 1-2° below the previous angle reached by the pendulum each time.

## Calculations

① Find the average angle reached by the pendulum. Record this value in table 8.1.

② Calculate the muzzle velocity of the projectile launcher.

**Table 8.1**

M= ______________

m= ______________

$R_{cm}$= ______________

| | |
|---|---|
| $\theta_1$ | |
| $\theta_2$ | |
| $\theta_3$ | |
| $\theta_4$ | |

Average θ= ______________

Muzzle Velocity= ______________

## Questions

① Is there another way to measure the muzzle velocity that you could use to check your results? You may want to use another method and compare the two answers.

② What sources of error are there in this experiment? How much do these errors affect your result?

③ It would greatly simplify the calculations (see theory section) if kinetic energy were conserved in the collision between ball and pendulum. What percentage of the kinetic energy is lost in the collision between ball and pendulum? Would it be valid to assume that energy was conserved in that collision?

④ How does the angle reached by the pendulum change if the ball is *not* caught by the pendulum? You may test this by turning the pendulum around so the ball strikes the back of the ball catcher. Is there more energy or less energy transferred to the pendulum?

# Ballistic Pendulum - Theory

## Overview

The ballistic pendulum is a classic method of determining the velocity of a projectile. It is also a good demonstration of some of the basic principles of physics.

The ball is fired into the pendulum, which then swings up a measured amount. From the height reached by the pendulum, we can calculate its potential energy. This potential energy is equal to the kinetic energy of the pendulum at the bottom of the swing, just after the collision with the ball.

We cannot equate the kinetic energy of the pendulum after the collision with the kinetic energy of the ball before the swing, since the collision between ball and pendulum is inelastic and kinetic energy is not conserved in inelastic collisions. Momentum is conserved in all forms of collision, though; so we know that the momentum of the ball before the collision is equal to the momentum of the pendulum after the collision. Once we know the momentum of the ball and its mass, we can determine the initial velocity.

There are two ways of calculating the velocity of the ball. The first method (approximate method) assumes that the pendulum and ball together act as a point mass located at their combined center of mass. This method does not take rotational inertia into account. It is somewhat quicker and easier than the second method, but not as accurate.

The second method (exact method) uses the actual rotational inertia of the pendulum in the calculations. The equations are slightly more complicated, and it is necessary to take more data in order to find the moment of inertia of the pendulum; but the results obtained are generally better.

Please note that the subscript "cm" used in the following equations stands for "center of mass."

## Approximate Method

Begin with the potential energy of the pendulum at the top of its swing:

$$\Delta PE = Mg\Delta h_{cm}$$

Where M is the combined mass of pendulum and ball, g is the acceleration of gravity, and Δh is the change in height. Substitute for the height:

$$\Delta h = R(1 - \cos\theta)$$

$$\Delta PE = MgR_{cm}(1 - \cos\theta)$$

Here $R_{cm}$ is the distance from the pivot point to the center of mass of the pendulum/ball system. This potential energy is equal to the kinetic energy of the pendulum immediately after the collision:

$$KE = \frac{1}{2} Mv_P^2$$

The momentum of the pendulum after the collision is just

$$P_p = Mv_P \ ,$$

which we substitute into the previous equation to give:

$$KE = \frac{P_P^2}{2M}$$

Solving this equation for the pendulum momentum gives:

$$P_p = \sqrt{2M(KE)}$$

This momentum is equal to the momentum of the ball before the collision:

$$P_b = mv_b \ .$$

Setting these two equations equal to each other and replacing KE with our known potential energy gives us:

$$mv_b = \sqrt{2M^2 gR_{cm}(1 - \cos\theta)}$$

Solve this for the ball velocity and simplify to get:

$$v_b = \frac{M}{m}\sqrt{2gR_{cm}(1 - \cos\theta)}$$

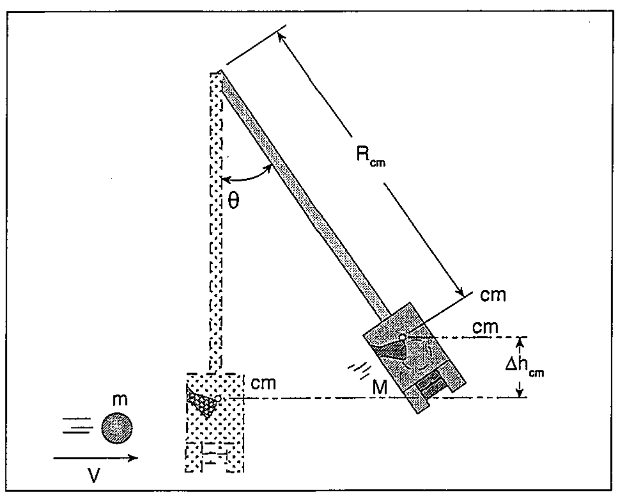


Figure 1